Institute for Energy Technology

Process and Fluid Flow Technology

A new drift-flux model for all gas volume fractions (Revision 2)

1 Introduction

In a previous document, we outlined a set of classical-type drift-flux models written in a form suitable for implementation in Oliasoft simulation software, including the WellTemp and WellKill modules. The model is briefly recaptured in section 2 here.

The classical drift-flux model for two-phase gas-liquid pipe flow describes the slip between the gas and liquid phases as the combined effect of non-uniform distribution of gas and liquid across the pipe cross section and the additional effect of gas buoyancy as well as local hydraulic gradients near the tip of a long bubble (Taylor bubble).

The main assumption in the model is that these two effects are additive:

$$U_G = C_0 U_M + U_d$$

Where U_G is the gas velocity and U_M the mixture velocity given by the sum of the gas and liquid superficial velocities

$$U_M = U_{SG} + U_{SL}$$

The **distribution coefficient** C_0 and the **drift velocity** U_d are often taken to be flow regime dependent, since the physical phenomena they reflect vary widely with the spatial distribution of gas and liquid.

Note that continuity implies that $U_{SG} = \alpha_G U_G$, $U_{SL} = (1-\alpha_G)U_L$ and $U_M = \alpha_G U_G + (1-\alpha_G)U_L$.

From the above we can infer

$$U_L = \frac{U_M - \alpha_G U_G}{1-\alpha_G} = \frac{1-\alpha_G C_0}{1-\alpha_G} U_M - \frac{\alpha_G}{1-\alpha_G} U_d$$

We also have

$$\alpha_G = \frac{U_{SG}}{C_0 U_M + U_d}$$

The classical-type drift-flux model of Bhagwat and Ghajar (2014) has been employed for two-phase gas-liquid flow, since this model has been developed to cover all pipe or well inclinations and thus seems to be the most general and up to data drift-flux model available. We have then added the three-phase corrections from Shi et al (2003) on top of the basic two-phase gas-liquid drift-flux model.

2 The standard drift-flux model for two-phase flow

Here we will first go through the two-phase drift-flux model of Bhagwat and Ghajar (2014), which is valid for all inclinations, and then add the three-phase corrections from Shi et al (2003) in the following section.

2.1 Distribution coefficient for gas-liquid slip

Bhagwat and Ghajar correlated the distribution parameter as follows:

$$C_0 = \frac{2 - \left(\frac{\rho_G}{\rho_L}\right)^2}{1 + \left(\frac{Re}{1000}\right)^2} + \frac{\left[(1 + \left(\frac{\rho_G}{\rho_L}\right)^2 cos\theta)/(1 + cos\theta)\right]^{\left(\frac{1-\alpha}{5}\right)} + C_{0,1}}{1 + \left(\frac{1000}{Re}\right)^2}$$

Where θ is the pipe inclination from the horizontalyj, and the mixture Reynolds number is given by

$$Re = U_M \rho_L D_H / \mu_L$$

The hydraulic diameter D_H is taken as the pipe diameter in pipe flow. In annulus flow, the hydraulic diameter is taken as the difference between inner and outer diameter, $D_H = D_{outer} - D_{inner}$.

The distribution parameter for horizontal flow is given by

$$C_{0,1} = C_1 \left(1 - \sqrt{\frac{\rho_G}{\rho_L}}\right) \left[(2.6 - \beta)^{0.15} - \sqrt{f_{tp}}\right] (1 - x)^{3/2}$$

except for gravity dominated flow ($-50° \leq \theta \leq 0°$ and $Fr_{SG} \leq 0.1$), where $C_{0,1}$ is set to zero. The constant C_1 is 0.2 for circular and annular cross sections and 0.4 for rectangular cross sections.

x is the so-called two-phase flow quality, which is equal to the flowing gas *mass* fraction:

$$x = \frac{\rho_G U_{SG}}{\rho_G U_{SG} + \rho_L U_{SL}}$$

β is the flowing gas *volume* fraction (GVF)

$$\beta = \frac{U_{SG}}{U_{SG} + U_{SL}}$$

The two-phase Fanning friction factor can then be computed using Churchill's friction formula with the two-phase Reynolds number and hydraulic diameter defined above:

$$f_{tp} = f_{Churchill}(Re, \varepsilon / D_H)$$

Remember that the Fanning friction factor f is 1/4 of the Darcy friction factor λ.

2.2 Drift velocity for gas-liquid slip

The drift velocity U_d (denoted U_{gm} in the paper of Bhagwat and Ghajar) is given by the following equations:

$$U_d = (0.35\, sin\theta + 0.45 cos\theta)\sqrt{(1-\alpha_G) g D_H \left(1 - \frac{\rho_G}{\rho_L}\right) C_2 C_3 C_4}$$

Where the coefficients C_2, C_3 and C_4 are given by

$$C_2 = \begin{cases} \left(\dfrac{0.434}{\log_{10}\left(\frac{\mu_L}{0.001}\right)}\right)^{0.15} & \text{for} \quad \dfrac{\mu_L}{0.001} > 10 \\ 1 & \text{for} \quad \dfrac{\mu_L}{0.001} \leq 10 \end{cases}$$

$$C_3 = \begin{cases} \left(\dfrac{La}{0.025}\right)^{0.9} & \text{for} \quad La < 0.025 \\ 1 & \text{for} \quad La \geq 0.025 \end{cases}$$

$$C_4 = \begin{cases} -1 & \text{for} \quad -50^\circ \leq \theta \leq 0^\circ \text{ and } Fr_{SG} \leq 0.1 \\ 1 & \text{otherwise} \end{cases}$$

The Laplace number La is given by

$$La = \sqrt{\frac{\sigma}{g(\rho_L - \rho_G) D_H^2}}$$

Here Fr_{SG} is the Froude number based on the gas phase, $Fr_{SG} = \frac{U_g}{\sqrt{\left(1-\frac{\rho_G}{\rho_L}\right) g D_H}}$.

2.3 Drift flux model for three-phase flow

In the original design, the three-phase drift-flux model of Shi et al (2003) was used. There are some limitations in this model regarding pipe inclination and other parameters that we want to avoid. We have therefore opted to combine the two-phase drift-flux model of Bhagwat and Ghajar with the oil/water slip model of Hasan and Kabir (1999) which is used by Shi et al. for their three-phase corrections. We take care to ensure that no inconsistencies arise from combining these two models.

2.4 From two-phase to three-phase

For the three-phase case, we first use the two-phase model to compute the gas and liquid velocities assuming known gas and liquid volume fractions and mixture velocity (transient case). Since we only will be using the three-phase model for dynamic kill simulations, we only need to consider the transient case here. We can thus assume that the gas, oil and water holdups are known from the previous time step or iteration.

As a logical analogy to the drift-flux relation for gas and liquid, we assume a quasilinear relation between the oil velocity U_O and the volume average liquid velocity U_L:

$$U_O = C_0' U_L + U_d'$$

Where we let the prime (‘) denote the distribution coefficient and drift velocity for the oil-water slip.

Once the oil velocity has been computed, the water velocity is then given by continuity. This gives

$$U_W = \frac{U_{SL} - \alpha_O U_O}{\alpha_W} = \frac{U_{SL} - \alpha_O U_O}{1 - \alpha_G - \alpha_O}$$

2.5 Distribution coefficient for oil-water slip

The oil-water distribution coefficient is then given by

$$C_0' = \begin{cases} A' & \text{for} \quad \alpha_O \leq B_1' \\ 1 & \text{for} \quad \alpha_O \geq B_2' \\ A' - (A' - 1)\dfrac{\alpha_O - B_1'}{B_2' - B_1'} & \text{for} \quad B_1' \leq \alpha_O \leq B_2' \end{cases}$$

Where the parameters A', B_1' and B_2' have the values 1.2, 0.4 and 0.7 respectively.

2.6 Drift velocity for oil-water slip

The drift velocity for the oil-water slip is given by

$$U_d' = 1.53 V_C' (1 - \alpha_O)^2 (cos\theta)^{0.5} (1 + sin\theta)^2$$

Where the characteristic velocity V_C' is given by

$$V_C' = \left[\frac{(\rho_W - \rho_O) g \sigma_{OW}}{\rho_W^2}\right]^{1/4}$$

3 Improved/revised drift-flux model

The classical drift-flux model is invalid as the gas volume fraction (GVF) approaches unity, and as a result, the liquid velocity may approach ±∞, and when $U_{SL} \rightarrow 0$, it can be easily shown that the gas velocity approaches infinity already as $\alpha_G \rightarrow 1/C_0$.

The deeper reason for this is that the drift-flux model is based on the physics of slug flow and/or bubbly flow, which can generally be classified as liquid dominated flow regimes (flow patterns). The form of the drift-flux equation is the same as the equation for the velocity of Taylor bubbles (long bubbles) in slug flow, and hence the behaviours of the two equations are similar under a wide range of conditions.

Since bubbly flow (or dispersed bubble flow) can be regarded as a special case of slug flow without long bubbles, the drift-flux equation can be adapted of bubbly flow by imposing a distribution parameter C_0 close to unity. Any dependence of the slip on pipe inclination and fluid properties will then enter through the drift velocity U_d.

To account for this limitation the classical drift flux model will be replaced by an alternative model for cases with a high GVF. In the classical drift-flux model, the gas velocity is expressed as a linear function of the *mixture* velocity. In the alternative model, the gas velocity is expressed as a linear function of the *liquid* velocity, allowing for greater flexibility of the gas and liquid velocities to adjust to each other under extreme conditions like for example very high gas volume fractions.

The normal situation in horizontal or upward flow, or in high velocity downward flow, is that the gas velocity is higher than the mixture velocity, which in turn is higher than the liquid velocity. Thus, the coefficients in the new, alternative (hybrid) drift-flux model will be different from those in the classical model, and/or the distribution coefficient C_0 must be higher in the new alternative (hybrid) model, since U_G/U_L should be higher than U_G/U_M in most cases.

It is conceivable that both the distribution coefficient C_0 and the drift velocity U_d might be higher in the new model than in the classical model; however, from functional analysis it may be argued that the constant term should be the same, since the velocity dependence is contained in the linear C_0 term. For the time being, we will assume that the drift velocity term will be carried over from the classical model to the new model.

Interpolation

The classical drift-flux model is very well-established and tested for liquid dominated flows, i.e. slug and bubbly flow. The classical drift-flux model was implemented in the kill module in Oliasoft's software, and should be kept more or less unchanged for liquid dominated flow. The new approach is to employ the new drift-flux model for gas dominated flow, and then use an interpolation between the two models for intermediate gas volume fractions. From experience we have defined gas dominated flow as multiphase flow with an input gas volume fraction of more than 90%. This may sound like a very strict definition, but dynamically it makes sense since a GVF of 90%

corresponds to a gas *mass* fraction of 50% at a gas density of about 90 kg/m³ (depending on the liquid density).

4 The new (hybrid) drift-flux model

4.1 Two-phase gas-liquid flow

The model equations for the new drift-flux models are written as follows:

$$U_G = C_1 U_L + U_d$$

The distribution coefficient C_1 has been tuned against an experimental data base consisting of approximately 4800 data points for horizontal and moderately inclined flow, 2400 points for vertical flow, and 660 points for steeply inclined flow. C_1 is given by

$$C_1 = C_H \cos^2\theta + C_V \sin^2\theta$$

Where

$$C_H = \begin{cases} 1 + k\beta^5 \left(\frac{\rho_L}{\rho_G}\right)^{0.85} \max\left(1, \frac{Fr_c}{Fr_d}\right)\cos\theta, & \theta \geq 0 \\ 1, & \theta < 0 \end{cases}$$

And

$$C_V = 1$$

Again, β is the gas volume fraction (GVF) and θ the inclination relative to the horizontal. The tuning constants are given by the coefficient k = 0.11 and critical Froude number $Fr_c = 1$.

We impose the upper limit $C_H \leq 5$, otherwise C_H could become very large for high density ratios ρ_L/ρ_G. The upper limit of 5 has been determined by tuning, but does not apply to atmospheric air/water data, which are frequently cited in the literature but irrelevant for our application.

The "densitometric" Froude number is given by

$$Fr_d = \frac{\rho_G U_{SG}^2}{(\rho_L - \rho_G) g D cos\theta}$$

The justification for the distinction between upflow ($\theta > 0$) and downflow ($\theta < 0$) is that the slip in near horizontal gas-liquid flow tends to be much higher for small to moderate upward inclinations than for small to moderate downward inclinations, except for very high mixture velocities, where the flow can be viewed as independent of gravity.

The drift velocity is kept the same as for the classical drift-flux model, since this coefficient is independent of the gas and liquid superficial velocities in the most common cases. The drift velocity is consequently computed from the classical Bendiksen correlation:

$$U_d = (0.54\cos\theta + 0.35\sin\theta)\sqrt{(1-\rho_G/\rho_L)gD}$$

For downward inclined flow we have to make a correction, since the tip of the Taylor bubble will switch direction for low velocities:

$$\theta < 0 \; and \; Fr < 1: \begin{cases} C_1 = 0.9 \\ U_d = (-0.54\cos\theta + 0.35\sin\theta)\sqrt{(1-\rho_G/\rho_L)gD} \end{cases}$$

Where Fr is a suitable Froude number, e.g.

$$Fr = U_M/\sqrt{(1-\rho_G/\rho_L)gD}$$

Now for the new drift flux equation, we can compute the void fraction (gas volume fraction) in the following way:

$$\text{Define } V = U_{SG} + C_1 U_{SL} + U_d$$

$$\alpha_G = \begin{cases} \dfrac{U_{SG}}{V}, U_d = 0 \\ \dfrac{V - \sqrt{V^2 - 4U_{SG}U_d}}{2U_d}, U_d \neq 0 \end{cases}$$

We can then trivially compute $\alpha_L = 1 - \alpha_G, U_G = \frac{U_{SG}}{\alpha_G}, U_L = \frac{U_{SL}}{\alpha_L}$.

4.2 Interpolation method

For a smooth but fast transition between the liquid dominated (low GVF) and gas dominated (high GVF) regions, a weighting based on a hyperbolic tangent function is applied:

$$\omega = \frac{1}{2}[1 - \tanh(15(\beta - 0.85))]$$

$$C_1 = \omega C_{1,liq} + (1-\omega)C_{1,gas}$$

$$U_d = \omega U_{d,liq} + (1-\omega)U_{d,gas}$$

Where β is the gas volume fraction and *liq* and *gas* denote the values for low GVF and high GVF respectively.

5 Implementation

In this section we discuss some implementation issues.

5.1 The steady state case

For the steady state case, we normally assume that the gas and liquid superficial velocities U_{SG} and U_{SL} are known and that the void fraction α_G is unknown. In this case we can compute the unknown void fraction and phase velocities in the standard drift-flux model as described previously.

5.2 The dynamic case

For the dynamic or transient case, we normally assume that the mixture velocity U_M and the void fraction α_G or liquid holdup $(1 - \alpha_G)$ are known. In this case, when using the standard drift-flux model we have simply

$$U_G = C_0 U_M + U_d$$

$$U_L = \frac{U_M - \alpha_G U_G}{1 - \alpha_G}$$

While for the hybrid drift-flux model for high GVF, we have to apply the equations from the previous section in a slightly modified manner.

References

1. Shi, H., Holmes, J. A., Durlofsky, L. J., Aziz, K., Diaz, L. R., Alkaya, B. and Oddie, G. (2003): Drift-Flux Modeling of Multiphase Flow in Wellbores. SPE 84228.
2. Bhagwat, S. M. and Ghajar, A. J. (2014): A flow pattern independent drift flux model based on void fraction correlation for a wide range of gas-liquid two-phase flow. Int. J. of Multiphase Flow 59, pp. 186-205.